III. *The Force of fired Gun-powder, and the initial Velocities of Cannon Balls, determined by Experiments; from which is alſo deduced the Relation of the initial Velocity to the Weight of the Shot and the Quantity of Powder.* By Mr. Charles Hutton, *of the Military Academy at* Woolwich. *Communicated by* Samuel Horſley, *LL.D. Sec. R. S.*

Read Jan. 8, 1778.

THESE experiments I made at Woolwich in the ſummer of the year 1775, aſſiſted by ſeveral able officers of the royal artillery at that place, and other ingenious gentlemen. The object of them was the determination of the actual velocities with which balls are impelled from given pieces of cannon, when fired with given charges of powder. Theſe experiments were made according to the method invented by Mr. ROBINS, and deſcribed in his treatiſe, intitled, New Principles of Gunnery, of which an account was printed in the Philoſophical Tranſactions for the year 1743. Before the diſcoveries of that ingenious gentleman very little progreſs had been made in the true theory of military projectiles. His book, however, contained ſuch important diſcoveries, that it was

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ſoon tranſlated into ſeveral of the languages on the continent, and the famous Mr. L. EULER honoured it with a very extenſive commentary in his tranſlation of it into the German language. That part of it hath always been particularly admired which relates to the experimental method of aſcertaining the actual velocities of ſhot, and in imitation of which were made the experiments related in this paper. Experiments in the manner of Mr. ROBINS were generally repeated by his commentators and others, with univerſal ſatisfaction, the method being ſo juſt in theory, ſo ſimple in practice, and altogether ſo ingenious, that it immediately gave the fulleſt conviction of its excellence, and of the abilities of its author. The uſe which that gentleman made of this invention was, to obtain the actual velocities of bullets experimentally, in order to compare them with thoſe which he computed *a priori* from his new theory, and thereby to verify the principles on which it is founded. The ſucceſs was fully anſwerable to his expectations, and left no doubt of the truth of his theory, when applied to ſuch pieces and bullets as he had uſed: but theſe were very ſmall, being only muſket balls of about one ounce weight; for, on account of the great ſize of the machinery neceſſary for ſuch experiments, Mr. ROBINS and other ingenious gentlemen had not ventured to extend their practice beyond bullets of that kind, and satisfied themselves with earnestly wishing for experiments to be made in a similar manner with balls of a larger sort. By the experiments in this paper I have endeavoured, in some degree, to supply this defect, having made them with small cannon balls of above twenty times the size, or from one pound to near three pounds weight. These are the only experiments that I know of which have been made with cannon balls for this purpose, although the conclusions to be deduced from such are of the greatest importance to those parts of natural philosophy which are dependent on the effects of fired gunpowder; nor do I know of any other practical method of ascertaining the initial velocities of military projectiles within any tolerable degree of the truth. The knowledge of this velocity is of the utmost consequence in gunnery: by means of it, together with the law of the resistance of the medium, every thing is determinable relative to that business; for, besides its being an excellent method of trying the strength of different sorts of powder, it gives us the law relative to the different quantities of powder, to the different weights of shot, and to the different lengths and sizes of guns. Besides these, there does not seem to be any thing wanting to determine any inquiry that can be made concerning the flight and ranges of shot, except the effects arising from the resistance of the medium.

Of the nature of the experiment, and of the machinery uſed in it.

The intention of the experiment is to diſcover the actual velocity with which a ball iſſues from a piece, in the uſual practice of artillery. This velocity is very great; from one thouſand to two thouſand feet in a ſecond of time. For conveniently eſtimating ſo great a velocity, the firſt thing neceſſary is to reduce it, in ſome known proportion, to a ſmall one. This we may conceive to be effected thus: ſuppoſe the ball, with a great velocity, to ſtrike ſome very heavy body, as a large block of wood, from which it will not rebound, ſo that they may proceed forward together after the ſtroke. By this means it is obvious, that the original velocity of the ball may be reduced in any proportion, or to any ſlow velocity which may conveniently be meaſured, by making the body ſtruck to be ſufficiently large; for it is well known, that the common velocity, with which the ball and block of wood would move forward after the ſtroke, bears to the original velocity of the ball only, the ſame ratio which the weight of the ball hath to that of the ball and block together. Thus then velocities of one thouſand feet in a ſecond are eaſily reduced to

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thoſe of two or three feet only; which ſmall velocity being meaſured by any convenient means, let the number denoting it be increaſed in the proportion of the weight of the ball to the weight of the ball and block together, and the original velocity of the ball itſelf will thereby be obtained. In theſe experiments, this reduced velocity is rendered very eaſy to be meaſured by a very ſimple and curious contrivance, which is this: the block of wood, which is ſtruck by the ball, is not left at liberty to move ſtraight forward in the direction of the motion of the ball, but it is ſuſpended, as the weight or bob of a pendulum, by a ſtrong iron ſtem, having a horizontal axis at top, on the ends of which it vibrates freely when ſtruck by the ball. The conſequence of this ſimple contrivance is evident: This large balliſtic pendulum, after being ſtruck by the ball, will be penetrated by it to a ſmall depth, and it will then ſwing round its axis and deſcribe an arch, which will be greater or leſs according to the force of the blow ſtruck; and from the ſize of the arch deſcribed by the vibrating pendulum, the velocity of any point of the pendulum itſelf can be eaſily computed; for a body acquires the ſame velocity by falling from the ſame height, whether it deſcend perpendicularly down, or otherwiſe; therefore, the length of the arch deſcribed, and of its radius, being given, its verſed ſine becomes known, which is the height perpendicularly deſcended by the correſponding point of the pendulum. The height deſcended being thus known, the velocity acquired in falling through that height becomes known from the common rules for the deſcent of bodies by the force of gravity; and this is the velocity of that point of the pendulum: this velocity of any known point whatever is then to be reduced to the velocity at the center of oſcillation, by the proportion of their radii or diſtances from the axis of motion; and the velocity of this center, thus obtained, is to be eſteemed the velocity of the whole pendulum itſelf; which being now given, that of the ball before the ſtroke becomes known from the given weights of the ball and pendulum. Thus then the menſuration of the very great velocity of the ball is reduced to the obſervation of the magnitude of the arch deſcribed by the pendulum, in conſequence of the blow ſtruck. This arch may be meaſured after various ways: in the following experiments it was aſcertained by meaſuring the length of its chord by means of a piece of tape or ſmall ribband, the one end of which was faſtened to the bottom of the pendulum, and the reſt of it made to ſlide through a ſmall machine contrived for the purpoſe, which will be hereafter deſcribed; for thus the length of the tape drawn

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out, was equal to the length of the chord of the arch defcribed by the bottom of the pendulum.

This defcription may convey a general idea of the nature and principle of the experiment; but befides the center of ofcillation and the weights of the ball and pendulum, the effect of the blow depends alfo on the place of the center of gravity and the point of impact: it will, therefore, be now neceffary to give a more particular defcription of the machine, and of the methods of finding the abovementioned requifites, and then inveftigate our general rule for determining the velocity of the ball, in all cafes, from them and the chord of the arch of vibration.

Of the particular defcription of the machine, and of the determination of the centers of gravity and ofcillation.

Tab. 1. Fig. 1. is a reprefentation of the machine ufed in the firft three courfes of experiments; and fig. 2. of that which was ufed in the other two. I fhall here defcribe the former of thefe, and afterwards take notice of the few particulars in which the other differs from it when I come to treat of the ufe of the latter.

The firft pendulum confifted of a block of found and dry elm, being nearly a cube of twenty inches long, which

which was faſtened to a ſtrong iron ſtem on the back part of it by ſcrew-bolts, having a thick iron axis at the top, whoſe ends were turned truly cylindrical, to roll pretty freely in ſockets made to receive them; the whole being ſupported by a four-legged ſtand of very ſtrong timber, which was firmly fixed in the ground. A is the face of the cube into which the balls were fired; by means of the blow it is made to ſwing round the axis BC, and the chord of the arch thereby deſcribed is meaſured by the tape DEF faſtened to the bottom of the wood at D, and ſliding with ſome ſlight friction through a little machine of braſs, fixed at E for that purpoſe, the tape being marked with inches and tenths, for the more eaſily meaſuring of the chord or part of it drawn through by the pendulum. The whole length of this pendulum, from the middle of the axis to the ribband at D, was $102\frac{1}{2}$ inches. The weight and the other dimenſions were taken each day when the experiments were made, and then regiſtered; and the manner of diſcovering the places of the centers of gravity and oſcillation was as follows:

To find the center of oſcillation, the pendulum was hung up, and made to vibrate in ſmall arcs, and the time of making two or three hundred vibrations was obſerved by a half-ſecond pendulum. Having thus obtained the

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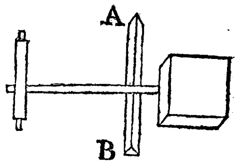
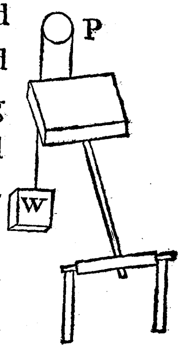
time anſwering to a certain number of vibrations, the finding of the center of oſcillation is eaſy: for if v denote the number of vibrations made in s ſeconds, then it is well known, that as $vv : ss :: 39.2 : \frac{39.2\, ss}{vv}$ = the diſtance in inches from the axis of motion to the center of oſcillation; and by this rule the place of that center was found for each day.

The center of gravity was aſcertained by one or both of the two following methods. Firſt, a triangular priſm of iron AB, being placed upon the ground with an edge upwards, the pendulum was laid acroſs it, and moved forward or backward on the ſtem or block as the caſe required, till the two ends exactly balanced each other; then, as it lay, the diſtance was meaſured from the middle of the axis to the part which reſted on the edge of the priſm, or the center of gravity of the pendulum. The other method was as in the latter of the two annexed figures, where the ends of the axis being ſupported on fixed uprights, and a chord faſtened to the lower end of the pendulum, was paſſed over a pulley at P, different weights W were faſtened to the other end of it, till the pendulum was





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brought to a horizontal poſition. Then, taking alſo the whole weight of the pendulum, and its length from the axis to the bottom where the chord was fixed, the place of the center of gravity is found by this proportion, as p the weight of the pendulum : w the appended weight :: d the whole length from the axis to the bottom : $\frac{dw}{p}$ = the diſtance from the axis to the center of gravity. Either of theſe two methods gave the place of the center of gravity ſufficiently exact; but the coincidence of the reſults of both of them was ſtill more ſatisfactory.

Of the rule for computing the Velocity of the ball.

Having deſcribed the methods of obtaining the neceſſary dimenſions, I proceed now to the inveſtigation of the theorem by which the velocity of the ball is to be computed. The ſeveral weights and meaſures being found, let then

b denote the weight of the ball,

p the whole weight of the pendulum,

g the diſtance of the center of gravity below the axis,

h the diſtance to the center of oſcillation,

k the diſtance to the point ſtruck by the ball,

z the velocity of this point ſtruck after the blow,

v the original velocity of the ball,

c the chord of the arch meaſured by the tape, and

r its radius, or the diſtance from the axis to the bottom of the pendulum.

Then the effect of the blow ſtruck by the ball is as $\frac{gh}{kk}$; or, as $kk : gh :: p : \frac{ghp}{kk}$ = the weight of a body, which, being placed at the point ſtruck, would acquire the ſame velocity from the blow as the pendulum does at the ſame point. Here then are two bodies, b and $\frac{ghp}{kk}$, the former of which, with the velocity v, ſtrikes the latter at reſt, ſo that after the blow they both proceed uniformly forward together with the velocity z; in which caſe it is well known, that $b : b + \frac{ghp}{kk} :: z : v$; and therefore the velocity z is $= \frac{bkkv}{bkk + ghp}$. But becauſe of the acceſſion of the ball to the pendulum, the place of the center of oſcillation will be changed; and from the known property of that point we find $\frac{bkk + ghp}{bk + gp}$ = to its diſtance from the axis. Call this diſtance of the center of oſcillation, of the maſs compounded of the ball and pendulum, H. Then, ſince z is the velocity of the point whoſe diſtance is k, we have this proportion, as $k : H :: z : \frac{zH}{k} = \frac{bkv}{bk + gp}$ = the velocity of this compound center of oſcillation.

Again,

Again, ſince $\frac{cc}{2r}$ is the verſed ſine of the deſcribed arc c, its radius being r; therefore as $r : H :: \frac{cc}{2r} : \frac{cc}{2rr} \times \frac{bkk+gbp}{bk+gp} =$ the verſed ſine to the radius H, or the verſed ſine of the arc deſcribed by the center of oſcillation, which call V; then is V the perpendicular height deſcended by this center, and the velocity it acquires by the deſcent through this ſpace is thus eaſily found, *viz.* as $\sqrt{16\frac{1}{12}} : \sqrt{V} :: 32\frac{1}{6} : \frac{8.02c}{r\sqrt{2}} \times \sqrt{\frac{bkk+gbp}{bk+gp}}$ = the velocity of the center of oſcillation deduced from the chord of the arc which is actually deſcribed.

Having thus obtained two different expreſſions for the velocity of this center, independent of each other, let an equation be made of them, and it will expreſs the relation of the ſeveral quantities in the queſtion; thus then we have $\frac{bkv}{bk+gp} = \frac{8.02c}{r\sqrt{2}}\sqrt{\frac{bkk+gbp}{bk+gp}}$ from which we obtain $v = \frac{8.02c}{bkr\sqrt{2}}\sqrt{\overline{bk+gp} \times \overline{bkk+gbp}}$ the true expreſſion for the original velocity of the ball the moment before it ſtruck the pendulum.

COROLLARY. But this theorem may be reduced to a form much more ſimple and fit for uſe, and yet be ſufficiently near the truth. Thus, let the root of the compound factor $\sqrt{\overline{bk+gp} \times \overline{bkk+gbp}}$ be extracted, and it will be equal to $\sqrt{b} \times \overline{pg+bk} \times \frac{b+k}{2h}$ within the 100000th part of of the truth in ſuch caſes as generally happen. But ſince $bk \times \frac{h+k}{2h}$ is uſually but about the 300th or 400th part of pg, and that bk differs from $bk \times \frac{h+k}{2h}$ but by about the 80th or 100th part of itſelf, therefore $pg+bk$ is within about the 20000th or 30000th part of $pg+bk \times \frac{h+k}{2h}$. Conſequently v is $= 8.02 c\sqrt{\tfrac{1}{2}h} \times \frac{pg+bk}{bkr}$ very nearly. Or, farther, if g be written for k in the laſt term bk, then finally v is $= 8.02 cg\sqrt{\tfrac{1}{2}h} \times \frac{p+b}{bkr}$, or $v = 5.672 cg\sqrt{h} \times \frac{p+b}{bkr}$, which is an eaſy theorem to be uſed on all occaſions; and being within about the 3000th part of the truth, it is ſufficiently exact for all practical purpoſes whatever. Where it muſt be obſerved, that c, g, k, r, may be taken in any meaſures, either feet or inches, &c. provided they be but all of the ſame kind; but h muſt be in feet, becauſe the theorem is adapted to feet.

SCHOLIUM. As the balls remain in the pendulum during the time of making one whole ſet of experiments, by the addition of their weight to it, both its weight and the centers of gravity and oſcillation will be changed by the addition of each ball which is lodged in the wood, and therefore p, g, and h, muſt be corrected after every ſhot in the theorem for determining the velocity v. Now the ſucceeding value of p is always $p+b$;

or

or p muſt be corrected by the continual addition of b: and g is corrected by taking always $g+\frac{k-g}{p+b}b$, or $g+\frac{k-g}{p}b$ nearly for each ſucceſſive value of g; or g is corrected by adding always $\frac{k-g}{p}b$ to the next preceding value of g: and laſtly, h is corrected by taking for its new values ſucceſſively $\frac{pgh+bkk}{pg+bk}$, or by adding always $\frac{k-h}{pg+bk}bk$, or $\frac{k-h}{p}b$ nearly, to the preceding value of h; ſo that the three corrections are made by adding always,

b to the value of p,

$\frac{k-g}{p} \times b$ to the value of g,

$\frac{k-h}{p} \times b$ to the value of h.

Before we proceed to the experiments it may not be improper to take notice of three ſeeming cauſes of error, which have not been brought to account in our theorem for determining the velocity of the ſhot; and to examine here whether their effects can ſenſibly affect the concluſion. Theſe are the penetration of the ball into the wood of the pendulum, the reſiſtance of the air to the back of it, and the friction on the axis: by each of theſe three cauſes the motion of the pendulum ſeems to be retarded. The principle on which our rule is founded ſuppoſes the momentum of the ball to be communicated to the pendulum in an inſtant; but this is not accurately the caſe, becauſe that this force is communicated during the ſmall time in which the ball makes the penetration; but as this is generally effected before the pendulum has moved one-tenth of an inch out of its vertical poſition, and uſually amounts to ſcarcely more than the 200th part of a ſecond, its effect will be quite imperceptible, and therefore it may ſafely be neglected in theſe experiments. As to the ſecond retarding force, or the reſiſtance of the air to the back of the pendulum, it is manifeſt that it will be quite inſenſible, when it is conſidered that its velocity is not more than three feet in a ſecond, that its ſurface is but about twenty inches ſquare, and that its weight is four or five hundred pounds. Neither can the effect of the laſt cauſe, or the friction on the axis, ever amount to a quantity conſiderable enough to be brought into account in theſe experiments: for, beſides that care was taken to render this friction as ſmall as poſſible, the effect of the little part which does remain is nearly balanced by the effect it has on the diſtance of the center of oſcillation; for as this center was determined from the actual vibrations of the pendulum, the friction on the axis would a little retard its motion, and cauſe its vibrations to be ſlower, and the conſequent diſtance of this center to be greater; ſo that the other parts of our theorem being multiplied by $\sqrt{h}$, or the root of this diſtance, which is as the time of a vibration, it is evident that the friction in the one caſe operates againſt that in the other; and that the difference of the two is the real efficacious cauſe of reſiſtance, and which therefore is either equal to nothing, or very nearly ſo.

Theſe general cauſes of error in the principles of the experiments are therefore ſafely omitted in the theorem: and our only care muſt be to guard againſt accidental errors in the actual execution of the buſineſs.

Of the experiments.

The gun, with which the experiments were made, was of braſs; the diameter of the bore or cylinder at the muzzle was $2\frac{4}{25}$, or 2.16 inches; but its diameter next the breech was a ſmall matter leſs, being there only $2\frac{2}{25}$, or 2.08 inches; ſo that the greateſt caſt-iron ball it would admit was juſt $19\frac{1}{2}$ ounces avoirdupois, or $1\frac{1}{4}$ pound wanting half an ounce; but ſometimes leaden balls were uſed, which weighed above $1\frac{3}{4}$ pound, and ſometimes long or cylindrical ſhot which weighed near three pounds; the length of the bore was $42\frac{3}{5}$, or 42.6 inches, ſo that it was nearly $20\frac{1}{2}$ calibers long.

The powder uſed was of the ſort which is commonly made for government; the quantity was two, four, or

 eight

eight ounces to a charge, which was always put into a light flannel bag, and rammed more or leſs, as expreſſed in each day's experiments, but without ever uſing any wad before it.

The diſtance of the gun from the pendulum was 29 or 30 feet; which diſtance was found by firing the piece, with eight ounces of powder without a ball, at different diſtances, till the force of the elaſtic fluid was found not to move the pendulum.

The penetrations of the balls into the wood were attempted to be taken, but were ſoon neglected on account of their uncertainty, becauſe of ſo many balls ſtriking in or near the ſame part of the wood. The depth of the penetration ſeemed to be near about three inches in ſolid wood when two ounces of powder was uſed.

The firſt courſe of experiments was on the 13th of May, 1775, it being a clear, dry day. The weights and meaſures then taken were thus, *viz.*

$p = 328$ pounds, the weight of the pendulum,

$g = 72$ inches, the diſtance of the center of gravity,

$h = 88$ inches $= 7\frac{1}{3}$ feet, the diſtance of the center of oſcillation,

$r = 102\frac{1}{2}$ inches, the diſtance to the bottom or tape.

The value of $h = 88$ was determined from the number of forty vibrations being made in a minute; for as

$40^2 : 60^2 :: 4 : 9 :: 39.2 : 88$. The number of ſhot was eight, and the circumſtances and reſults as exhibited in the following table.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, *k*.	Weight of the ball.	Weight of the ball, *b*.	Values of *p*.	Values of *g*.	Chord of the arc, *c*.	Veloc. *per* ſecond.
	Oz.	In.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet.
1	2	1.98		92.5	$17\frac{1}{2}$	1.094	328.0	72.0	13.0	458
2	2	1.98		92.5	$17\frac{1}{2}$	1.094	329.1	72.1	17.8	631
3	2	1.98	3.15	91.6	$17\frac{1}{2}$	1.094	330.2	72.2	18.1	650
4	2	1.97	3.15	91.	$17\frac{1}{4}$	1.078	331.3	72.3	17.6	646
5	2	1.97	3.15	90.5	$17\frac{1}{4}$	1.078	332.3	72.3	16.3	604
6	2	1.96	3.15	92.4	17	1.063	333.4	72.4	16.2	598
7	4	1.97	4.5	92.	$17\frac{1}{4}$	1.078	334.4	72.5	24.0	881
8	4	1.96	4.5	90.5	17	1.063	335.5	72.5	25.0	950

By computing the velocities from our theorem inveſtigated in the corollary, they come out as they are here regiſtered in the laſt column of the table, and they are all pretty regular excepting the firſt one, which is about one-fourth part leſs than the reſt with the ſame weight of powder, and which irregularity muſt have been cauſed by ſome unperceived accident. The values of *p* and *g* were each corrected by their reſpective theorems; but the value of *b* was kept the ſame ($7\frac{1}{3}$ feet) through-

 out,

out, becaufe that its correction was fo fmall as not to make a difference of above a foot or two at moft in the velocity: and for the fame reafon this correction is neglected, as quite unneceffary, in the reft of the experiments of the other days following.

The mean velocity of the fecond, third, fourth, fifth, and fixth numbers is 626, and of the feventh and eighth it is 915; that is, the velocity with two ounces of powder was 626 feet *per* fecond, and that with four ounces was 915 feet; and thefe two velocities are in the ratio of 1 to 1.46. But the mean weight of the balls in the former cafe was $17\frac{1}{3}$ ounces, and in the latter it was $17\frac{1}{8}$ ounces; and the ratio of the quantities of powder was that of 1 to 2. But the direct fub-duplicate ratio of the powder, compounded with the inverfe fub-duplicate ratio of the weights of the fhot, forms the ratio of 1 to 1.42, which is nearly equal to the ratio (1 to 1.46) of the velocities; that is, in this inftance the velocities are very nearly as the fquare roots of the quantities of powder directly, and the fquare roots of the weights of the balls inverfely. The powder was forced up with only one ftroke of the rammer.

The fecond courfe was performed on the 3d of June, 1775, which was a clear, dry day, but windy. Some of the

the experiments of this day are doubtful, as indeed is evident from their irregularity, on account of the wind blowing the tape, which was not very properly fecured by the little brazen machine through which it was made to flide.

The powder was taken from the bottom of a barrel, and the charges rammed a little clofer than thofe of the former day; and fo tight did the fhots fit towards the breech, that many ftrokes of the rammer were neceffary to drive them home.

The fourth and fifth fhots were of a long form, which may be called fpherico-cylindrical, as they were cylinders terminated by hemifpherical ends, fo that their fection through the axis was of this form ▭, and the length of the axis was near double the diameter of the fhot.

The fourth fhot, or firft of the long fort, ftruck fideways, making a hole of the fhape of the above fection, only its length or axis was not horizontal but vertical, thus ▯.

The laft fhot lay obliquely in the wood; it appeared to have ftruck with its end foremoft, or nearly fo, as the oblique pofition in which it lay feemed to be caufed by its ftriking againft a former fhot lodged in the wood, with the hance of its end, fo as to flatten it in that part.

Of

Of the pendulum, the weight, length, and centers of gravity and of oſcillation were the ſame as when taken the former day before the experiments were made; the former balls having been extracted, and the holes filled up with wood.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, *k*.	Weight of the ball.	Weight of the ball, *b*.	Values of *p*.	Values of *g*.	Chord of the arc, *c*.	Veloc. *per* ſecond.
	Oz.	In.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet.
1	2	2.08	2.85	88½	19½	1.219	328.0	72.0	24.3	800
2	2	2.08	2.85	89	19½	1.219	329 2	72.1	30.5	1003
3	2	2.08	2.85	93½	19½	1.219	330.4	72.1	30.0	943
4	2	2.08	3 35	92½	46½	2.906	331.6	72.2	57 0	767
5	2	2.08	3.35	93	46½	2.906	334.5	72.4	54.0	731

Here the firſt ſhot is again ſo much ſmaller than the two following ones, that ſome irregularity muſt have attended it, on which account we cannot make any uſe of it. The mean between the ſecond and third is 973; and between the fourth and fifth the mean is 749; that is, the velocity of the 19½ ounce ball is 973, and that of the 46½ ounce ſhot 749 feet *per* ſecond, which two numbers are in the ratio of 1.3 to 1. But the reciprocal ſubduplicate ratio of the weights (19½ and 46½) is the ratio of 1.54 to 1: therefore, in this inſtance, the velocity of the heavier ſhot is a little leſs than would ariſe from the

inverſe

inverſe ratio of the ſquare roots of the weights of the ſhot. But the accurate ratio cannot certainly be drawn from theſe numbers, on account of the doubtfulneſs of ſome of them, as was before obſerved.

It is very remarkable, that in the experiments of this day, the mean velocity with two ounces of powder is 973, whereas it was no more than 626 in the former day with the ſame quantity of powder, notwithſtanding the balls were heavier with the greater velocity in the proportion of 19 to 17 nearly. This remarkable difference muſt be chiefly owing to the windage in the firſt courſe: and from hence we may perceive the great advantage to be gained by the uſe of balls approaching in proportion nearer to the diameter of the bore of the gun than what is preſcribed in the preſent eſtabliſhment. Poſſibly, however, ſome part of this difference might be owing to ſome ſmall inequality in the powder, as that which was uſed this day was taken from the bottom of a barrel. Perhaps alſo ſome part of the effect may be owing to the greater degree of ramming which the powder had in this courſe.

The third courſe was made on the 12th of June, 1775, it being a clear, dry, and calm day. The powder in the experiments of this day was rammed in the ſame

degree

degree as in the laſt one. It was alſo nearly the ſame in the ſucceeding days, as may be perceived by inſpecting the fourth column of each courſe, which, denoting the height of the charge, ſhews the degree of compactneſs with which the powder was lodged in the piece. The dimenſions, as taken this day, were thus:

$p = 324$ pounds, the weight of the pendulum.

$g = 71.4$ inches, the diſtance of the center of gravity.

$h = 88$ inches $= 7\frac{1}{3}$ feet, the diſtance of the center of oſcillation.

$r = 102\frac{1}{2}$ inches, the whole length to the tape.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, *k*.	Weight of the ball.	Weight of the ball, *b*.	Values of *p*.	Values of *g*.	Chord of the arc, *c*.	Veloc. *per* ſecond.
	Oz.	Inches.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet
1	2	2.080	2.85	94	$19\frac{1}{2}$	1.219	324.0	71.4	23.0	700
2	2	2.036	2.85	94	$18\frac{1}{4}$	1.141	325.2	71.5	24.5	799
3	2	2.045	2.85	$93\frac{1}{2}$	$18\frac{1}{2}$	1.156	326.4	71.6	22.0	715
4	4	2.062	4.	$92\frac{1}{4}$	19	1.188	327.5	71.7	27.3	880
5	4	2.036	4.	$93\frac{1}{2}$	$18\frac{1}{4}$	1.141	328.7	71.7	35.0	1163
6	4	2.045	4.	$93\frac{1}{2}$	$18\frac{1}{2}$	1.156	329.9	71.8	33.0	1087

Here the common mean weight of the ball is $18\frac{2}{3}$ ounces, the mean velocity with two ounces of powder is 738, and that with four ounces of powder is 1043 feet *per* ſecond. The ratio of theſe two velocities is that of

1 to

1 to 1.414; that is, accurately the ratio of the ſquare roots of the quantities of powder.

Of the experiments made with the other pendulum, which is repreſented in fig. 2.

The firſt pendulum was gradually more and more rent and ſhattered by the firing of ſo many balls into it, till at the end of the laſt courſe of experiments it had become quite uſeleſs. Another was then fitted up, and with it were performed the two following courſes.

This ſecond pendulum conſiſted of a cubical block of ſound elm, of near two feet long, fixed to the iron ſtem, but not exactly in the manner of the former; for in this the ſtem was placed vertically over the center of the top-end, to which point it continued whole, but there divided in two, each paſſing to right and left over the top down the ſides, and returning along the bottom, and being at proper intervals faſtened to the wood with iron pins. A thick ſheet of lead was faſtened over each of the two upright faces into which the ſhot were to be fired, both to guard them from ſplintering very much, and to add to the weight of the pendulum. The whole was then firmly ſecured by two very thick iron bands or hoops, paſſed horizontally quite around the wood, and

 firmly

firmly fixed to it, the one next the upper end, and the other near the lower, ſo as ſtrongly to reſiſt the endeavours of the ſhot to ſplit the wood.

The whole weight of the pendulum, thus fitted up, was 552 pounds; its whole length, from the middle of the axis to the tape at the bottom, was 101 inches; the diſtance to the center of gravity was 78 inches; and the diſtance to the center of oſcillation was 88 inches equal to $7\frac{1}{3}$ feet, which was exactly the ſame as that of the former pendulum, their numbers of vibrations being alike in the ſame time.

Inſtead of ſuſpending this pendulum, after the manner of the former, by the ends of its axis in grooves turned to fit them, they were only placed on flat, level pieces of wood, on which this pendulum vibrated much freer than the other did; but a ſmall nail was driven into the ſupporting wood, juſt behind each end of the axis, to prevent the ſtroke of the ſhot from throwing it off the ſtand.

To this pendulum was adapted a better machine for the tape to ſlide through than the former one was, the inconvenience of which had often been experienced by its catching and entangling the tape, ſo as to interrupt its free motion, and once indeed to break it. This new one, however, is at once very ſimple, and perfectly free from

every

every inconvenience, giving juſt the neceſſary degree of friction to the tape, without ever ſtopping its motion; ſo that of the real quantity drawn out by the vibration of the pendulum there could not poſſibly be the leaſt doubt. This ſimple contrivance conſiſted barely of about ſix or eight inches of the liſt of woollen cloth faſtened upon the arch of a ſmall piece of wood, which was ſhaped into the form of the ſegment of a circle thus , the tape being made to paſs through between the curved ſide and the liſt, which was moderately ſtretched and faſtened by its two ends to thoſe of the little arch.

Upon the whole, the machinery was all ſo perfect, and every circumſtance attending the experiments of the two enſuing days ſo carefully obſerved, that I can with great ſafety rely on the concluſions reſulting from them. And as thoſe of the one day were made with leaden balls, and thoſe of the other with iron ones, which differ greatly in weight, every other circumſtance being the ſame, they afford very good means for diſcovering the law of the different weights of ſhot, while the variations in the powder from two to four and eight ounces furniſh us with the rule for the different quantities of it.

The fourth courſe was on the 20th of July, a fine clear day. The powder was a mixture of ſeveral of the ſorts made for government, and the balls were of lead. The quantities of powder were two, four, and eight ounces alternately; and the dimenſions at firſt were thus:

p = 552 pounds, the whole weight of the pendulum.

r = 101 inches, its whole length.

g = 78 inches, the diſtance of the center of gravity.

h = 88 inches = $7\frac{1}{3}$ feet, that of the center of oſcillation.

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, k.	Weight of the ball.	Weight of the ball, b.	Values of p.	Values of g.	Chord of the arc c.	Veloc. *per* ſecond.
	Oz.	Inches.	Inches.	Inches.	Oz.	Pounds	Pounds	Inches.	Inches.	Feet
1	2	2.021	2.85	90.	$28\frac{1}{4}$	1.766	552.0	78.0	14.8	612
2	4	2.021	4.4	87.	$28\frac{1}{4}$	1.766	553.8	78.0	20.5	879
3	8	2.032	7.1	87.	$28\frac{3}{4}$	1.797	555.5	78.1	27.5	1164
4	2	2.026	2.85	90.	$28\frac{1}{2}$	1.781	557.3	78.1	15.0	622
5	4	2.026	4.4	88.	$28\frac{1}{2}$	1.781	559.1	78.1	20.5	871
6	8	2.032	7.1	92.	$28\frac{3}{4}$	1.797	560.9	78.2	28.5	1154
7	2	2.021	2.85	89.8	$28\frac{1}{4}$	1.766	562.7	78.2	14.3	605
8	4	2.026	4.4	91.3	$28\frac{1}{2}$	1.781	564.5	78.2	21.0	870
9	8	2.026	7.1	87.	$28\frac{1}{2}$	1.781	566.2	78.3	26.8	1169

Let us now collect together the ſeveral velocities belonging to the ſame quantity of powder, in order to take their means, thus:

Veloc.

	Veloc. with 2 ounces.	Veloc. with 4 ounces.	Veloc. with 8 ounces.
	612	879	1164
	622	871	1154
	605	870	1169
	3)1839	3)2620	3)3487
The means,	613	873	1162

The uniformity of thefe velocities is very ftriking, and the means with two, four, and eight ounces of powder are 613, 873, and 1162, which are in the ratio of 1, 1.424, and 1.9; thefe numbers are nearly in the ratio of the fquare roots of the quantities (2, 4, and 8) of powder, the numbers in this latter ratio being 1, 1.414, and 2, where the fmall difference lies chiefly in the laft number. A fmall part of this defect in the greateft velocity is to be attributed to the mean weight of the balls ufed with it being greater than in the others; for the mean weight of the balls ufed with eight ounces of powder is $28\frac{2}{3}$ ounces, while that with the two and four ounces is only $28\frac{1}{3}$; the reciprocal fub-duplicate ratio of thefe is that of 1 to 1.006, in which proportion, increafing 1.9 the number for the greater velocity, it becomes 1.91, which ftill falls fhort of 2 by .09, which is about the $\frac{1}{22}$d part too fmall for the fub-duplicate ratio of the

the powder. This defect of a 22d part is owing to three evident caufes, *viz.* 1. The lefs length of cylinder through which the ball was impelled; for by infpecting the fourth column, denoting the height of the charge, it appears, that the balls lay three or four inches nearer to the muzzle of the piece with the eight ounce charge than with the others. 2. The greater quantity of elaftic fluid which efcaped in this cafe than in the others by the windage; this happens from its moving with a greater velocity, in confequence of which a greater quantity efcapes by the vent and windage than with the fmaller velocities. 3. The third caufe is the greater quantity of powder blown out unfired in this cafe than in that of the lefs velocities; for the ball which was impelled with the greater velocity would be fooner out of the piece than the others, and the more fo as it had a lefs length of the bore to move through; and if powder fire in time, which cannot be denied, although indeed that time is manifeftly very fhort, a greater quantity of it muft remain unfired when the ball with the greater velocity iffues from the piece, than when that which has the lefs velocity goes out, and ftill the more fo as the bulk of powder which was at firft to be inflamed in the one cafe fo much exceeded that in the others. The effect, however, will arife chiefly from the firft and laft of of theſe three cauſes, as that of the ſecond will amount to very little; becauſe that the effect ariſing from the greater velocity with which the fluid eſcapes at the vent and windage, is partly balanced by the ſhorter time in which it acts.

From the above reflections we may alſo perceive, how ſmall the quantity of powder is which is blown out unfired in any of theſe caſes, and the amazing quickneſs with which it fires in all caſes: for although the time in which the ball paſſed through the barrel, when impelled by the eight ounces of powder, was not greatly different from the half only of the time in which it was impelled by the two ounces, it is evident that in half the time there was nearly four times the quantity of powder fired.

The fifth or laſt courſe was on the 21ſt of September, 1775, fine clear weather, but a little windy.

The machinery and the balls were of iron, but powder the ſame as in the laſt courſe, and the dimenſions as follows:

$p = 553$ pounds, the weight of the pendulum.

$r = 101$ inches, its length.

$g = 78\frac{1}{8}$ inches, the diſtance of the center of gravity.

$h = 84.775$ inches $= 7.065$ feet, that of the center of oſcillation, the pendulum making 68 vibrations in 100 ſeconds.

Number

Number.	Weight of powder.	Diam. of the ball.	Height of the charge.	Struck below the axis, *k*.	Weight of the ball.		Weight of the ball, *b*.	Values of *p*.	Values of *g*.	Chord of the arc, *c*.	Velo. *per* second.
	Oz.	Inches	Inches	Inches.	Oz.	Dr.	Pounds	Pounds	Inches.	Inch.	Feet.
1	2	2.062	3.	88.3	19	0	1.188	553.0	78.1	11.4	702
2	4	2.062	4.3	88.3	19	0	1.188	554.2	78.1	17.3	1068
3	8	2.062	6.7	91.0	19	0	1.188	555.5	78.2	23.6	1419
4	2	2.070	3.	90.7	19	$3\frac{1}{2}$	1.201	556.8	78.2	11.4	682
5	4	2.080	4.3	90.7	19	$8\frac{1}{2}$	1.221	558.1	78.2	17.3	1020
6	8	2.064	6.7	90.7	19	$0\frac{3}{4}$	1.190	559.4	78.2	22.3	1352
7	2	2.060	3.	91.	18	15	1.184	560.6	78.3	11.4	695
8	4	2.058	4.3	90.	18	14	1.180	561.9	78.3	15.3	948
9	8	2.049	6.7	90.	18	$9\frac{3}{4}$	1.163	563.1	78.3	22.9	1443
10	2	2.047	3.	88.3	18	9	1.160	564.3	78.3	10.9	703
11	4	2.037	4.3	88.3	18	$4\frac{1}{3}$	1.142	565.5	78.4	14.8	973
12	8	2.036	6.7	88.3	18	$3\frac{1}{4}$	1.140	566.6	78.4	20.6	1360
13	2	2.034	3.	92.	18	3	1.137	567.8	78.4	11.4	725
14	4	2.034	4.3	92.	18	3	1.137	569.0	78.4	15.0	957
15	8	2.031	6.7	94.3	18	$1\frac{1}{2}$	1.131	570.1	78.5	22.5	1412

Let us now take the means among thofe of the fame quantity of powder, thus:

Veloc.

	Veloc. with 2 ounces.	Veloc. with 4 ounces.	Veloc. with 8 ounces.
	702	1068	1419
	682	1020	1352
	695	948	1443
	703	973	1360
	725	957	1412
	5)3507	5)4966	5)6986
The means,	701	993	1397

And these mean velocities with two, four, and eight ounces of powder, are as the numbers 1, 1.416, and 1.993; but the sub-duplicate ratio of the weights (two, four, and eight) of powder gives the numbers 1, 1.414, and 2, to which the others are sufficiently near. It is obvious, however, that the greatest difference lies in the last number which answers to the greatest velocity, and which is again in defect. It will still be a little more in defect if we make the allowance for the weights of the balls; for the mean weight of the balls with the two and four ounces is $18\frac{3}{4}$ ounces, but of the eight ounces it is $18\frac{3}{5}$; diminishing therefore the number 1.993 in the reciprocal sub-duplicate ratio of $18\frac{3}{5}$ to $18\frac{3}{4}$, it becomes 1.985, which falls short of the number 2 by ,015 or the 133d part of itself; which defect is to be

 attri-

attributed to the ſame cauſes as it was in the laſt courſe of experiments before explained.

Let us now compare the correſponding velocities in this courſe and the laſt.

In this courſe they are	701, 993, 1397;
In the laſt they were	613, 873, 1162.

Now the ratio of the firſt two numbers, or the velocities with two ounces of powder, is that of 1 to 1.1436; the ratio of the next two, is that of 1 to 1.1375; and the ratio of the laſt is that of 1 to 1.2022. But the mean weight of the ſhot was, for two and four ounces of powder $28\frac{1}{3}$ ounces in the laſt courſe, and $18\frac{3}{4}$ ounces in this; and for eight ounces of powder, it was $28\frac{2}{3}$ in the laſt, and $18\frac{3}{5}$ in this: taking now the reciprocal ſubduplicate ratios of theſe weights of ſhot, we obtain the ratio of 1 to 1.224 for that of the balls which were fired with two ounces and four ounces of powder, and the ratio of 1 to 1.241 for the balls which were fired with eight ounces. But the real ratios above found are not greatly different from theſe. And the variation of the actual velocities from this law of the weights of ſhot incline the ſame way in this courſe, as they appeared to do in the ſecond courſe of theſe experiments.

We may now collect into one view the principal inferences that have resulted from these experiments.

1. And first, it is made evident by them, that powder fires almost instantaneously, seeing that almost the whole of the charge fires though the time be much diminished.

2. The velocities communicated to balls, or shot of the same weight, with different quantities of powder, are nearly in the sub-duplicate ratio of those quantities. A very small variation, in defect, taking place when the quantities of powder become great.

3. And when shot of different weights are fired with the same quantity of powder, the velocities communicated to them are nearly in the reciprocal sub-duplicate ratio of their weights.

4. So that, universally, shot which are of different weights, and impelled by the firing of different quantities of powder, acquire velocities which are directly as the square roots of the quantities of powder, and inversely as the square roots of the weights of the shot, nearly.

5. It would therefore be a great improvement in artillery to make use of shot of a long form, or of heavier matter; for thus the momentum of a shot, when fired

with the ſame weight of powder, would be increaſed in the ratio of the ſquare root of the weight of the ſhot.

6. It would alſo be an improvement to diminiſh the windage; for by ſo doing, one-third or more of the quantity of powder might be ſaved.

7. When the improvements mentioned in the laſt two articles are conſidered as both taking place, it is evident that about half the quantity of powder might be ſaved, which is a very conſiderable object. But important as this ſaving may be, it ſeems to be ſtill exceeded by that of the article of the guns; for thus a ſmall gun may be made to have the effect and execution of one of two or three times its ſize in the preſent mode, by diſcharging a ſhot of two or three times the weight of its natural ball or round ſhot. And thus a ſmall ſhip might diſcharge ſhot as heavy as thoſe of the greateſt now made uſe of.

Finally, as the above experiments exhibit the regulations with regard to the weights of powder and balls, when fired from the ſame piece of ordnance, &c.; ſo by making ſimilar experiments with a gun, varied in its length, by cutting off from it a certain part before each courſe of experiments, the effects and general rules for the different lengths of guns may be certainly determined by them. In ſhort, the principles on which theſe

expe-

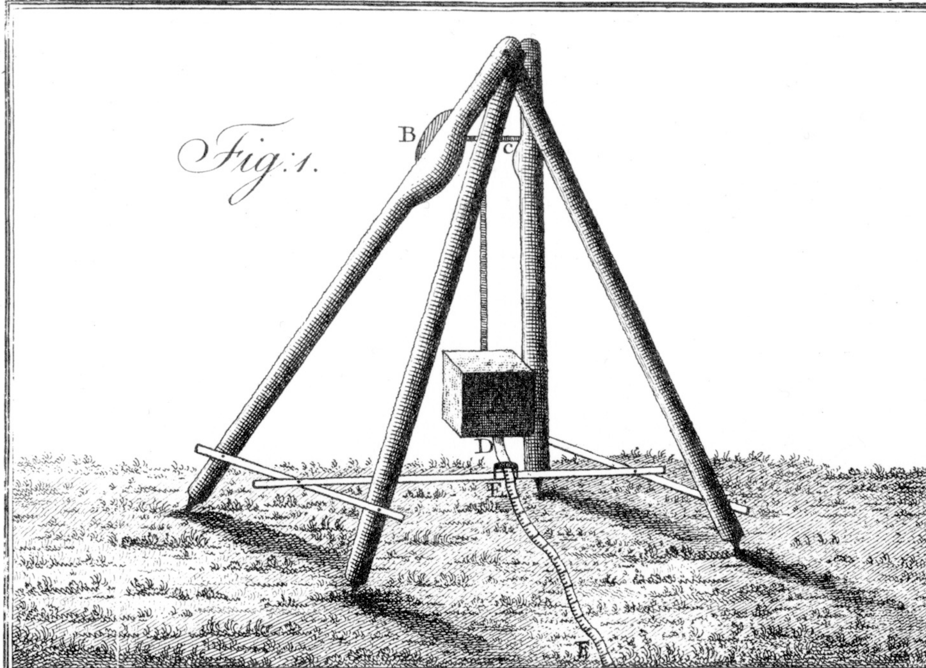

Fig: 1.

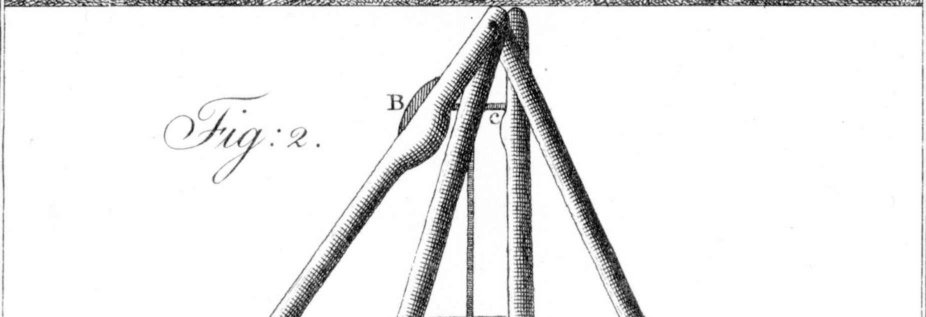

Fig: 2.

Basire Sculp:

experiments were made, are ſo fruitful in conſequences, that, in conjunction with the effects reſulting from the reſiſtance of the medium, they ſeem to be ſufficient for anſwering all the enquiries of the ſpeculative philoſopher, as well as thoſe of the practical artilleriſt.